Embedding the proficiency strands in teaching and learning primary school mathematics

Session 2

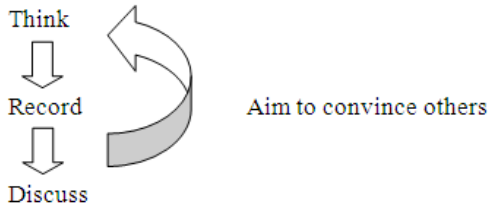
Tell me about the answer!

Tell me "about" the answer

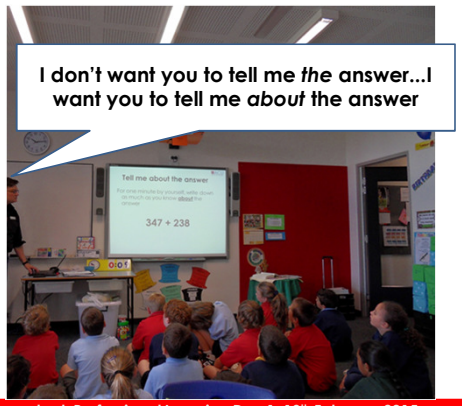
For one minute by yourself, write down as much as you know **about** the answer

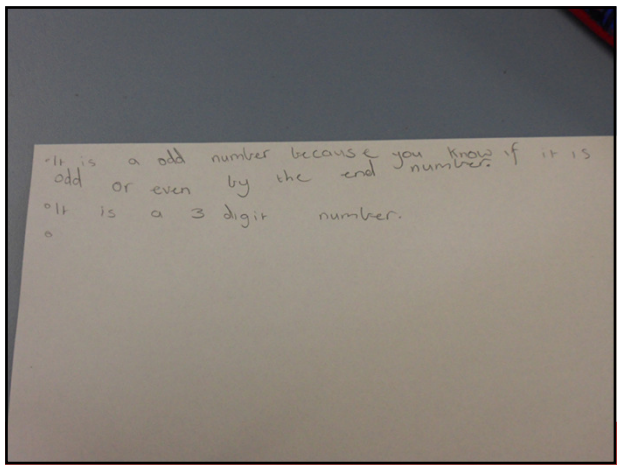
347 + 238

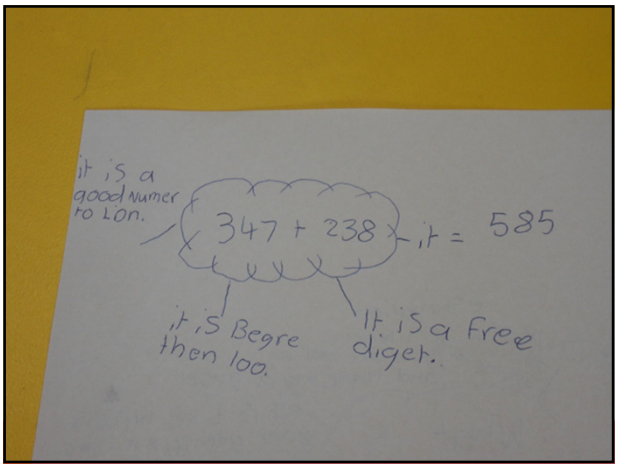
Think, record, discuss cycle

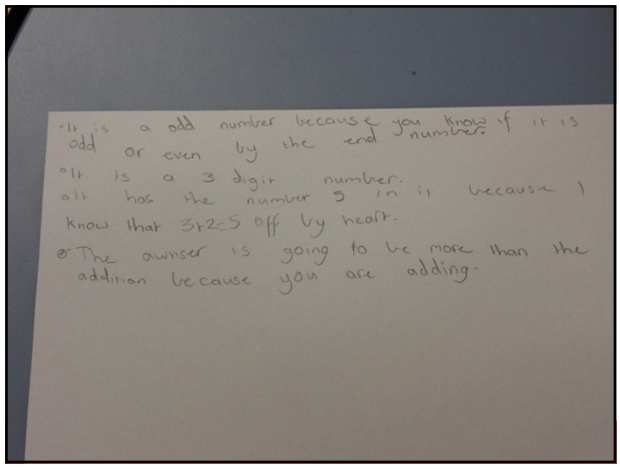


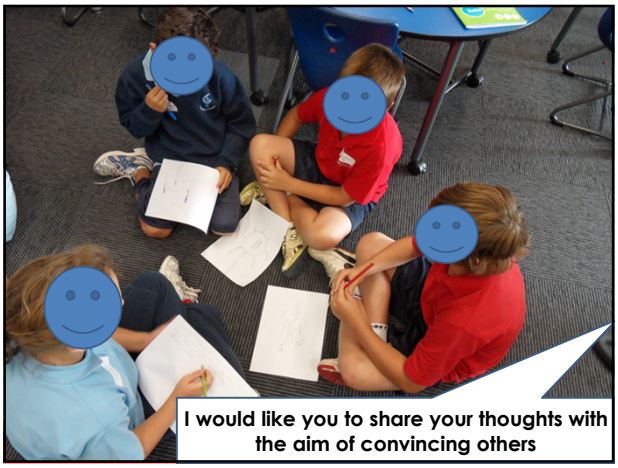
(Brown, 2008)

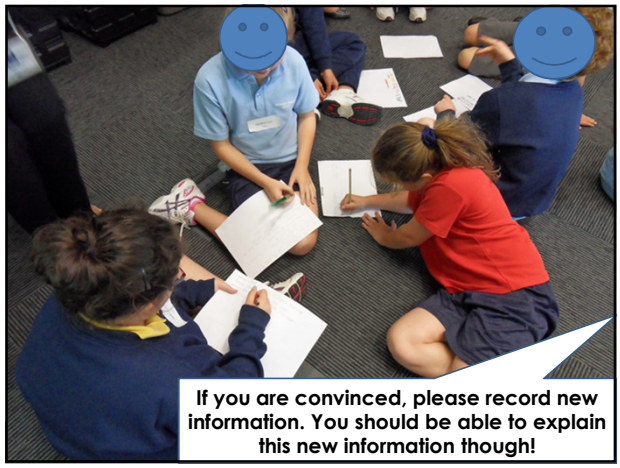


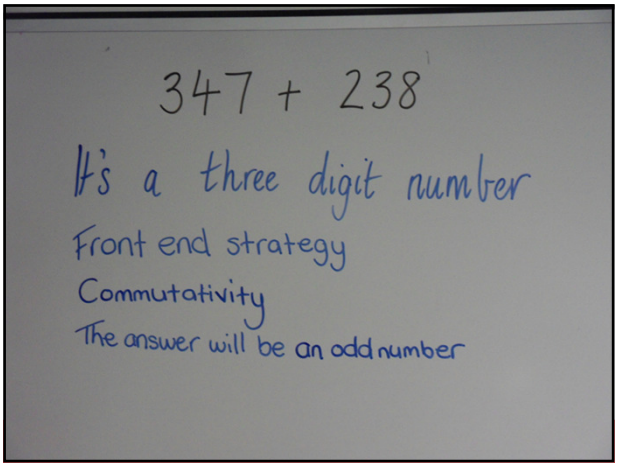


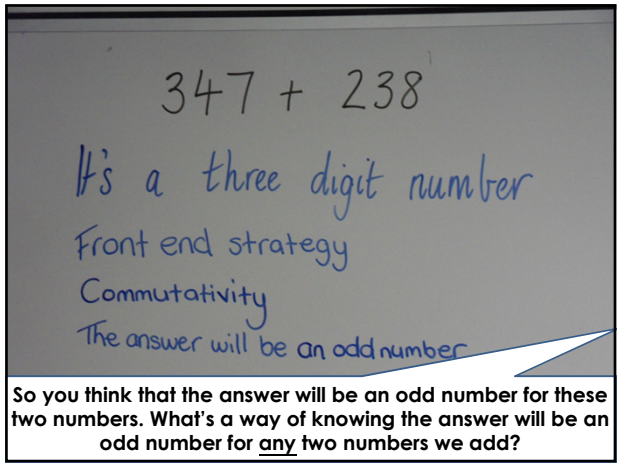





I would like you to share your thoughts with the aim of convincing others


If you are convinced, please record new information. You should be able to explain this new information though!





So you think that the answer will be an odd number for these two numbers. What's a way of knowing the answer will be an odd number for any two numbers we add?

Tell me *about* the answer

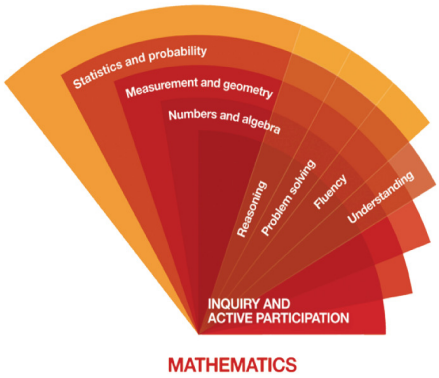
2.67 × 4.7

1227 – 567

2⅝ + 1⅓

$2768 ÷ 37

ACARA (2010)



Why the proficiencies?

Ultimately, the proficiencies (the ways of working and thinking mathematically) are taught to develop the students' **understanding**, learning, use and enactment of concepts, skills, and dispositions towards mathematics

Understanding mathematics

There are two types of understanding:

- **instrumental understanding** - possessing a rule and the ability use it 'rules without reasons
- **relational understanding** - 'knowing both what to do and why'

(Skemp, 1976)

Understanding mathematics

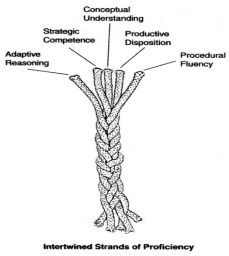
- Understanding is the measure of quality and quantity of connections between new ideas and existing ideas
- Knowing ≠ Understanding (students may know something about fractions, for example, but not understand them)
- Richard Skemp named the ends of the continuum of understanding

Relational understanding
What to do and why

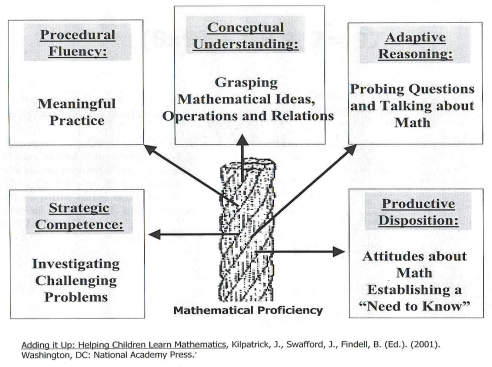
Instrumental understanding
Just doing it

Proficiency strands

Kilpatrick, Swafford & Findell (2001)



Kilpatrick, Swafford & Findell (2001)	Australian Curriculum equivalent
Conceptual understanding	Understanding
Procedural fluency	Fluency
Adaptive reasoning	Reasoning
Strategic competence	Problem solving
Productive disposition	



Adding it Up: Helping Children Learn Mathematics, Kilpatrick, J., Swafford, J., Findell, B. (Ed.). (2001). Washington, DC: National Academy Press.'

Intertwined strands of proficiency

- Conceptual **understanding** - comprehension of mathematics concepts, operations and relationships
- Procedural **fluency** - skill in carrying out procedures flexibly, accurately and appropriately
- Strategic competence – ability to formulate, represent and solve mathematical problems
- Adaptive **reasoning** – capacity for logical thought, reflection, explanation, and justification

Kilpatrick, Swafford & Findell (2001)

Integral to learning

- The proficiency strands of Understanding, Fluency, Problem Solving and Reasoning are **an integral part of mathematics** content across the three content strands: Number and Algebra, Measurement and Geometry, and Statistics and Probability.
- The proficiencies **reinforce the significance of working mathematically** within the content and describe how the content is explored or developed. They **provide the language** to build in the developmental aspects of the learning of mathematics.

(ACARA, 2012)

AC and the proficiencies

The proficiency strands articulate the mathematical ways of working and thinking in which students can engage when learning and using mathematical content.

Teachers emphasise, highlight and bring to attention the mathematical ways of working and thinking through the proficiencies.

(ACARA, 2011)

UNDERSTANDING	FLUENCY
Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.	Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions.
PROBLEM SOLVING	**REASONING**
Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.	Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.

UNDERSTANDING	FLUENCY
Students build a **robust knowledge** of adaptable and transferable mathematical concepts. They **make connections** between related concepts and progressively apply the familiar **to develop new ideas**. They develop an **understanding** of the **relationship** between the 'why' and the 'how' of mathematics. Students build understanding when they **connect related ideas**, when they **represent concepts in different ways**, when they **identify commonalities** and **differences** between aspects of content, when they **describe** their **thinking mathematically** and when they **interpret mathematical information**.	Students develop skills in **choosing** appropriate **procedures**, **carrying out procedures flexibly**, **accurately**, **efficiently** and **appropriately**, and **recalling** factual **knowledge** and concepts readily. Students are fluent when they **calculate answers efficiently**, when they recognise robust ways of answering questions, when they **choose** appropriate **methods** and **approximations**, when they **recall definitions** and **regularly use facts**, and when they can **manipulate expressions** and **equations** to find solutions.
PROBLEM SOLVING	**REASONING**
Students develop the ability to **make choices**, **interpret**, **formulate**, **model** and **investigate** problem situations, and **communicate solutions** effectively. Students **formulate** and **solve problems** when they use mathematics to represent unfamiliar or meaningful situations, when they **design investigations** and **plan** their **approaches**, when they **apply** their existing **strategies** to seek solutions, and when they **verify** that their **answers** are reasonable.	Students develop an increasingly sophisticated capacity for logical thought and actions, such as **analysing**, **proving**, **evaluating**, **explaining**, **inferring**, **justifying** and **generalising**. Students are **reasoning mathematically** when they **explain** their **thinking**, when they **deduce** and **justify strategies** used and conclusions reached, when they adapt the known to the unknown, when they **transfer learning from one context to another**, when they **prove** that something is true or false and when they **compare** and **contrast** related **ideas** and **explain** their **choices**.

Possible proficiencies for 'Tell me about the answer'

Understanding
- Knowing when the sum will be even or odd when adding two numbers
- Recognising that with addition of whole numbers the sum is larger than the addends
- Understanding place value system (100s, 10s, 1s)
- Interpreting 3-digit numbers in terms of understanding their quantity
- Visualising the quantities associated with the numerals

Fluency
- Using rounding strategies to provide approximations
- Recalling basic addition facts

Problem solving
- Extending number facts using place value knowledge
- Recording responses in ways that are mathematically appropriate

Reasoning
- Justifying possible responses providing convincing argument

Questions to consider

- What do we want students to learn when it comes to the proficiencies?
- How do we know if they have learned these proficiencies?
- What do we do if they don't know aspects of those proficiencies?
- What do we do if they already know those proficiencies?

Practical strategies

As a table group, what are some potential strategies for supporting the:
- teachers' use of the proficiencies in their practice
- students' use of the proficiencies to support their learning?

Learning intentions

Using the proficiencies as learning intentions during mathematics lessons

"Today we are focusing on **explaining** our mathematical thinking and **justifying** the strategies that we use"

Convince me!

Simply saying '**Convince me!**' can lead to opportunities for students to go beyond just the answer, moving to interrogating their own thinking whilst enacting the proficiencies

The overarching theme of 'convince me' leads to the notion of proof, an essential element of mathematical thinking and working mathematically

(Reiss, Heinze, Renkl, & Groβ, 2008)

Articulating understanding

When asking a student to share his/her understanding from a lesson, encourage the student to share what he/she used to think when articulating their new understanding

Before today, I used to think that...but now I think...

I think this because of my mathematical way of working that involved…

Important ideas

- Not all topics that students learn lend themselves to all of the proficiencies at the one time
- Plan and audit the proficiencies that are addressed through the term/year
- Highlight the proficiencies with students during the lesson and during reflection sessions

Important ideas

- Plan proficiencies as 'key skills' that could be used for teaching and assessment purposes
- View the proficiencies as vital as the content; we interact with and learn the ideas by using the proficiencies (they are not an "extra"!)

Personal reflection

How could you influence your classroom teachers in paying greater attention to teaching and assessing students' knowledge and use of the AC Proficiencies?